Department of ECE

Probability Theory and Stochastic Processes

B.Tech MID Examination Question Bank (R22)

Semester - III

PART-A

MID-I Questions					
Q.No.	**Questions**	**Marks**	**BL**	**CO**	**Unit**
1	Define random experiment.	2 M	L1	CO1	1
2	Define sample space and classify.	2 M	L1	CO1	1
3	Define event and list various events types.	2 M	L1	CO1	1
4	Give classical and empirical definitions of probability.	2 M	L3	CO1	1
5	In a survey conducted by a news agency, 60% read Telugu newspaper, 40% read English newspaper and 20% read both. If a person is chosen at random and if he already reads English newspaper find the probability that he also reads Telugu newspaper.	2 M	L3	CO1	1
6	The sample space for an experiment is S = {0, 1, 2.5, 6}. List all possible values of the following random variable: (i) $X = 2s$ (ii) $X = 5s^2 - 1$	2 M	L3	CO1	1
7	DefineExpected value and list its properties.	2 M	L2	CO2	2
8	A random variable X have values -4, -1, 2, 3 and 4, each with probability $\frac{1}{5}$. Compute the mean.	2 M	L3	CO2	2
9	Define variance and state its properties.	2 M	L1	CO2	2
10	Describe Moment Generating Function and state any two properties.	2 M	L2	CO2	2
11	State and explain Central Limit Theorem.	2 M	L2	CO2	2
12	Compute the probability of the event $X \leq 5.5$ for gaussian random variable having $a_X = 2$and $\sigma_X = 2$.	2 M	L3	CO2	2
13	Define Stochastic Process and classify.	2 M	L1	CO3	3
14	Define Nth order stationary and strict sense stationary random processes.	2 M	L1	CO3	3
15	Determine if the constant process $X(t) = A$ is mean ergodic, where A is a random variable with mean $\bar{A}$ and variance σ^2.	2 M	L4	CO3	3
MID-II Questions					
16	Prove $\lvert R_{XX}(\tau)\rvert \leq R_{XX}(0)$	2 M	L4	CO3	3

17	If $X(t)$ and $Y(t)$ are two independent random variables with zero means, get the ACF of $Z(t) = a + bX(t) + cY(t)$.	2M	L3	CO3	3
18	Aircrafts arrive at an average rate of 6 per hour according to a Poisson process. Determine the probability of 5 arrivals in a span of 30 minutes.	2 M	L3	CO3	3
19	Define PSD and CPSD and give corresponding expressions.	2 M	L1	CO4	4
20	Determine which of the following can be and cannot be valid power density spectrums. (i) $e^{-(\omega-1)^2} (ii)\ 3 + j\omega^2$.	2 M	L3	CO4	4
21	Obtain PSD and compute the average power of a random process X(t) with ACF $R_{XX}(\tau) = \frac{A^2}{2}\cos(\omega_0\tau)$	2 M	L3	CO4	4
22	A random process $X(t)$ has $S_{XX}(\omega) = \frac{1}{25+\omega^2}$. Derive its autocorrelation function.	2 M	L3	CO4	4
23	How do you compute the average power of a random process, given its ACF and PSD?	2 M	L1	CO4	4
24	State Wiener Khinchine relations.	2 M	L1	CO4	4
25	Define Shot noise and Thermal noise.	2 M	L1	CO5	5
26	List the classification of noise.	2 M	L2	CO5	5
27	Explain the trade-off between bandwidth and SNR.	2 M	L2	CO5	5
28	Define bit rate and information rate.	2 M	L1	CO5	5
29	Describe source coding and list algorithms used for it.	2 M	L2	CO	5
30	State channel capacity theorem.	2 M	L1	CO5	5

PART-B

MID-I Questions					
No.	**Questions**	**Marks**	**BL**	**CO**	**Unit**
1	State addition theorem of probability. One integer is chosen at random from the integers 1, 2, 3 ...100. What is the probability that the chosen integer is divisible by 6 or 8?	4 M	L3	CO1	1
2	State multiplication theorem of probability. An ordinary 52-card deck is thoroughly shuffled. You are dealt three cards up in sequence. What	4 M	L3	CO1	1

<table>
<tr><td></td><td>is the probability that all three cards sevens?</td><td></td><td></td><td></td><td></td></tr>
<tr><td>3</td><td>State total probability theorem. A bag contains 2 white and 4 green balls. Another bag contains 3 white and 3 green balls. A bag is selected at random and a ball is drawn from it. What is the probability the ball is white?</td><td>4 M</td><td>L3</td><td>C1O 1</td><td>1</td></tr>
<tr><td>4</td><td>Define random variable and classify. State the conditions for a function to be a random variable.</td><td>4 M</td><td>L3</td><td>CO1</td><td>1</td></tr>
<tr><td>5</td><td>Define Density Function and state its properties.</td><td>4 M</td><td>L1</td><td>CO1</td><td>1</td></tr>
<tr><td>6</td><td>Define Distribution Function and list the properties.</td><td>4 M</td><td>L1</td><td>CO1</td><td>1</td></tr>
<tr><td>7</td><td>a) State and Prove Baye’s Theorem.
b) Given three identical boxes I, II, and III, each containing six coins. Box I contains three gold coins, two silver coins and one copper coin. Box II contains one gold coin, three silver coins and two copper coins. Box III contains two gold coins, one silver coin and three copper coins. A person chooses a box at random and takes out a coin.If the coin is gold, what is the probability that it was from box III?</td><td>8M</td><td>L3</td><td>CO1</td><td>1</td></tr>
<tr><td>8</td><td>a) Determine the constant b > 0 so that the function given below is a valid probability density.
$f_X(x) = \begin{cases} \frac{e^{3x}}{4} & 0 \le x \le b \\ 0 & elswhere \end{cases}$
b) Obtain distribution function of the random variable.</td><td>8M</td><td>L3</td><td>CO1</td><td>1</td></tr>
<tr><td>9</td><td>A discrete random variable X has the following probability table
<table>
<tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>P(X)</td><td>0</td><td>K</td><td>2K</td><td>2K</td><td>3K</td><td>K^2</td><td>$2 K^2$</td><td>$7 K^2+K$</td></tr>
</table>
(i) Obtain the value of k
(ii) CDF of X</td><td>8M</td><td>L3</td><td>CO1</td><td>1</td></tr>
<tr><td>10</td><td>Compute variance of a random variable X with density function
$$f_X(x) = \begin{cases} \frac{x}{6} & 2 < x \le 4 \\ 0 & elswhere \end{cases}$$</td><td>4M</td><td>L3</td><td>CO2</td><td>2</td></tr>
<tr><td>11</td><td>Two statistically independent random variables X and Y have means E[X] = 2, E[Y] =4. They have second moments $E[X^2] = 8$, and $E[Y^2] = 25$. Calculate the variance of $W = 3X - Y$.</td><td>4 M</td><td>L3</td><td>CO2</td><td>2</td></tr>
</table>

12	The joint density function of two random variables X and Y is as given below $f_{XY}(x,y) = \begin{cases} \frac{xy}{9} & 0 < x < 2 \ and \ 0 < y < 3 \\ 0 & elswhere \end{cases}$ Show that X and Y are uncorrelated.	4M	L4	CO3	2
13	Define Characteristic Function and list its properties. Derive MGF and CF for a binomial distributed random variable.	4 M	L4	CO2	2
14	Sketch the density of W = X + Y , if X and Y are independent with $f_X(x) = 0.4\delta(x-1) + 0.6\delta(x-2) \& f_Y(y) = 0.7\delta(y-5) + 0.3\delta(y-6)$	4 M	L3	CO2	2
15	Write a brief note on Gaussian distribution. List the properties of Gaussian random variables.	4 M	L3	CO2	2
16	Determine the following parameters for a uniformly distributed random variable. (i) $\bar{X}$ (ii) σ_X^2 (iii) $M_X(v)$ (iv) $\phi_X(\omega)$	8 M	L3	CO2	2
17	Compute the following parameters for a Poisson random variable. (i) Mean (ii) Variance (iii) Moment Generating Function (iv) Characteristic Function	8M	L3	CO3	2
18	Zero-man Gaussian random variables X_1, X_2 and X_3 having a covariance matrix $[C_X] = \begin{bmatrix} 3.0 & 1.8 & 1.1 \\ 1.8 & 3.0 & 1.8 \\ 1.1 & 1.8 & 3.0 \end{bmatrix}$ are transformed to new variables $Y_1 = 5X_1 + 2X_2 - X_3$ $Y_2 = -X_1 + 3X_2 + X_3$ $Y_3 = 2X_1 - X_2 + 2X_3$ (i) Derive the covariance matrix of Y_1, Y_2 and Y_3. (ii) Compute the variances of Y_1, Y_2 and Y_3. (iii) List the covariances between Y_1, Y_2 and Y_3. (iv) Write an expression for the joint density function of Y_1, Y_2 and Y_3.	8 M	L4	CO2	2

19	When do you call a random process as WSS&JWSS? Explain. A random process$X(t) = A_0 cos(\omega_0 t + \theta)$ where θ is a uniform random variable on (0,2π). Test whether $X(t)$ is wide sense stationary.	4M	L2	CO3	3
20	State Ergocity theorem, and state the conditions for meanergodic and correlation ergodic random processes? Given $X(t) = 2Acos(\omega_0 t + 2\theta)$ is a random Process, where 'θ' is a uniform random variable, over (0,2π). Check the process for mean ergodicity.	4M	L4	CO3	3
21	Given the random processes X(t) = Acosω_0t + Bsinω_0t where ω_0 is a constant, and A and B are uncorrelated zero mean random variables having different density functions but the variance σ^2, show that $X(t)$ is WSS.	4M	L4	CO3	3
	MID-II Questions				
22	Define autocorrelation function, state all its properties and prove any four.	4 M	L1	CO3	3
23	Define cross correlation function, state all its properties and prove any four.	4 M	L1	CO3	3
24	Assume that an ergodic random process X(t) has an autocorrelation function $R_{XX}(\tau) = 18 + \frac{2}{6+\tau^2}[1 + 4\cos(12\tau)]$. (i) Compute$\lvert\bar{X}\rvert$. (ii) Does this process have a periodic component? (iii) What is the average power in $X(t)$? (iv) Determine the variance of X(t).	4M	L3	CO3	3
25	Prove $S_{\dot{X}\dot{X}}(\omega) = \omega^2 S_{XX}(\omega)$	4 M	L3	CO4	4
26	A wide sense stationary random process has an autocorrelation function $R_{XX}(\tau) = Ke^{-3\lvert\tau\rvert}$, where K is a constant. Show that its power spectral density is given by $$S_{XX}(\omega) = \frac{2}{1 + \left(\frac{\omega}{K}\right)^2}.$$	4 M	L3	CO4	4
27	Find the crosscorrelation function corresponding to the power density spectrum $S_{XY}(\omega) = \frac{6}{(9+\omega^2)(3+j\omega)^2}$.	4 M	L3	CO4	4
28	Show that $S_{XY}(\omega) = S_{YX}(\omega) = 2\pi\bar{X}\bar{Y}\delta(\omega)$, if $X(t)$ and $Y(t)$ are uncorrelated.	4 M	L3	CO4	4

29	Derive ACF for the random process X(t) having PSD $S_{XX}(\omega) = A; \quad -k < \omega \leq k.$	4M	L3	CO4	4
30	If X(t) is a stationery process find the power spectrum of $Y(t) = a + bX(t)$in terms of PSD of X(t) if a and b are real constants.	4 M	L3	CO4	4
31	Define power spectral density of a random process, state and prove its properties.	8M	L4	CO4	4
32	Define cross power spectral density, state and prove its properties.	8M	L4	CO4	4
33	Derive the relationship between power spectral density and autocorrelation of a random process $X(t)$.	8 M	L4	CO4	4
34	An amplifier has 3 dB noise figure. Calculate the equivalent input noise temperature.	4M	L3	CO5	5
35	The noise present at the input of a two port network is 5 μW. The noise figure is 0.5 dB. The gain is 10^6. Calculate Output available noise power.	4M	L3	CO5	5
36	For an AWGN channel with 4 kHz bandwidth, the noise spectral density $\frac{N_0}{2}$is 1 $PicoWatts/Hz$ and the signal power at the receiver is 0.1 mW. Determine the capacity of the channel.	4M	L3	CO5	5
37	Define entropy and state its properties. A DMS has an alphabet of eight letters $x_i,\ i = 1,2,3,8$ with probabilities $0.25, 0.2, 0.15, 0.12, 0.1, 0.08, 0.05, 0.05$.Determine the entropy of the source.	4M	L3	CO5	5
38	Define Mutual information and state its properties.	4M	L2	CO5	5
39	A DMS consists of six $symbols\ x1,\ x2,\ x3,\ x4,\ x5\ and\ x6$ with probabilities 0.25, 0.3, 0.2, 0.12, 0.08, and 0.05 respectively. Apply Shannon's coding to find efficiency and redundancy.	4M	L3	CO5	5
40	a) Explain with necessary expressions the Effective Noise Temperature and Average Noise Figure of cascaded networks. b) Two 2 port devices are connected in cascade. First stage noise figure and gain are 15db and 42 respectively. For second stage noise figure and gain are 25 db and 35 respectively. Compute overall noise figure.	8M	L3	CO5	5
41	Define and explain narrow band noise in detail. List all its properties.	8M	L2	CO5	5

42	List the steps in Huffman source coding algorithm. A discrete memory less source consists of $symbols\ x1,\ x2,\ x3,\ x4,\ x5, x6, x7,$ and $x8$ with probabilities 0.2, 0.25, 0.2, 0.1, 0.1, 0.05, 0.05, and 0.05 respectively. Determine efficiency of Huffman coding.	8M	L3	CO5	5